P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010
Reaccredited at ‘A+’ level by NAAC
Autonomous & ISO 9001:2015 Certified

Title of the Course: REAL ANALYSIS - I
Semester : I

Course Code	23MA1T1	Course Delivery Method	Blended Mode
Credits	5	CIA Marks	30
No. of Lecture Hours / Week	5	Semester End Exam Marks	70
Total Number of Lecture Hours	75	Total Marks	100
Year of Introduction : 2020-2021	Year of offering : 2023-2024	Year of Revision: 2023-24	Percentage of Revision :20%

Course Objective: The main objective of this course is to develop problem solving skills and knowledge on the basic concepts of continuity, differentiation, Riemann-Stieltjes integrals, Improper integrals.

Course Outcomes: After successful completion of this course, students will be able to

CO1: understand the properties of continuous functions. (PO1)

CO2: understand the properties of differentiable functions. (PO4)

CO3: test the Riemann- Stieltjes integrability of bounded functions and their properties.(PO3)

CO4: understand the effect of uniform convergence on the limit function with respect to continuity, differentiability and integrability.(PO5)

CO5: test the convergence of improper integrals. (PO4)

UNIT-I

Continuity: Limits of functions- continuous functions- Continuity and Compactness- Continuity and Connectedness- Discontinuities.
[4.1 to 4.27 of chapter 4 of Text Book1]

UNIT-II

Differentiation:
Derivative of a Real Function- Mean value theorems- The Continuity of Derivatives- L' Hospital's rule- Derivatives of higher order- Taylor's theorem.
[5.1 to 5.15 of chapter 5 of Text Book1]

UNIT-III

The Riemann - Stieltjes Integral: Definition and Existence of Integral-Properties of the integral -Integration and Differentiation –Integration of vector-valued functions - Rectifiable Curves. [Chapter-6 of Text Book-1]

UNIT-IV

Sequences and Series of functions: Discussion of main problem - Uniform convergence – Uniform convergence and continuity – Uniform Convergence and Integration – Uniform Convergence and Differentiation – Equicontinuous families of functions – The Stone - Weierstrass Theorem.[7.1 to 7.26 of Text Book 1]

UNIT-V

Improper Integrals: Introduction – Integration of unbounded Functions with Finite limits of Integration – Comparison Tests for Convergence at a of $\int_a^b f dx$ - Infinite range of Integration – Integrand as a Product of Functions. [Chapter-11 of Text Book-2]

PRESCRIBED BOOKS:

1. Walter Rudin, "**Principles of Mathematical Analysis"**, Student Edition 1976, McGraw-Hill International Publishers.
2. S.C. Malik and Savita Aurora, " **Mathematical Analysis",** Fourth edition, New Age International Publishers.

REFERENCE BOOK:

1.Tom. M. Apostol, "**Mathematical Analysis"** second Edition, Addison Wesley Publishing Company.

Course has Focus on : Foundation

Websites of Interest: 1. www. nptel.ac.in
2. www.epgp.inflibnet.ac.in
3. www.ocw.mit.edu

P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA

(An Autonomous college in the jurisdiction of Krishna University)

M. Sc. Mathematics

First Semester

REAL ANALYSIS -23MA1T1

Time: 3 Hours **Max. Marks : 70**

SECTION- A

Answer all questions. **(5 X 4=20)**

1 (a) Let $f(x) = (1/x), x \neq 0$
$= 0, x = 0.$
Examine the continuity of the function f(x) on R. (CO1, L2)

(OR)

(b) If f is a continuous mapping of a compact metric space X into a metric space Y, then prove that f(X) is closed and bounded. (CO1, L2)

2 (a) Prove that every differentiable function on (a, b) is continuous on (a, b). (CO2, L1)

(OR)

(b) State and prove mean value theorem. (CO2, L1)

3 (a) Show that $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$. (CO3, L2)

(OR)

(b) State and prove fundamental theorem of calculus. (CO3, L2)

4 (a) Differentiate Pointwise convergence and Uniform convergence of sequence of functions. (CO4, L3)

(OR)

(b) For every interval [a, -a], prove that there is a sequence of real polynomials P_n such that $P_n(0) = 0$ and $\lim_{n\to\infty} P_n(x) = |x|$ uniformly on [a, -a] (CO4, L3)

5 (a) Examine the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x}}$ (CO5, L4)

(OR)

(b) Examine the convergence of $\int_0^\infty \sin x dx$ (CO5, L4)

SECTION- B

Answer all questions. All questions carry equal marks. **(5X10=50)**

6 (a) Show that a mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(V)$ is open in X, for every open set V in Y. (CO1, L3)

(OR)

(b) If f is a continuous mapping of a compact metric space X into a metric space Y, and E is a connected subset of X, then prove that f(E) is connected. (CO1, L3)

7 (a) Suppose f is continuous on [a, b], $f^1(x)$ exists at $x \in [a, b]$, g is defined on an interval I which contains the range of f, and g is continuous at f(x). If h(t) =g(f(t)), then prove that h is differentiable at x and $h^1(t) = f^1(g(x))g^1(x)$. (CO2, L2)

(OR)

(b) State and Prove Taylor's theorem. (CO2, L2)

8 (a) If f is monotonic on [a, b] and if α is continuous on [a, b] then show that $f \in R(\alpha)$. (CO3, L3)

(OR)

(b) If γ^1 is continuous on [a, b] then show that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma^1(t)| dt$. (CO3, L3)

9 (a) If $\{f_n\}$ is sequence of continuous functions on E and if $f_n \to f$ uniformly on E, then show that f is continuous on E. (CO4, L4)

(OR)

(b) State and prove Stone – Weierstrass theorem. (CO4, L4)

10(a) Test the convergence of the integral $\int_0^1 \frac{dx}{(x-a)^n}$ for n<1. (CO5, L4)

(OR)

(b) Show that if f and g are positive and $f(x) \leq g(x)$, for all x in [a, X] and $\int_a^\infty g(x)dx$ converges, then $\int_a^\infty f(x)dx$ converges. (CO5, L4)

***